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A Survey on Applications of Sampling Operators in Image Processing

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Abstract

The objective of this paper is to present a brief study of linear and non-linear cases of univariate and multivariate Kantorovich sampling operators and their applications to image processing. This kind of operators are very suitable to study applications to Signal and Image Processing. Image processing through sampling operators plays a crucial part in the diagnosis of Arterial diseases. The visual and empirical assessment of cardiac structure and function is an important challenge faced by medical elds and hospitals.

Keywords: *Sampling operators, Kantorovich operator, Image Processing, Orlicz Space.*

Introduction

Sampling Kantorovich Operators has been treated together with applications to image processing. It becomes interesting, not only from a mathematical point of view, but also in terms of applications. Kantorovich version of the linear generalized sampling type series is of the form

$$S_w f(x) = \sum_{k=-\infty}^{k=\infty} \chi\left(wx - t_k\right)\left[\frac{w}{\Delta_k}\int_{\frac{t_k}{w}}^{\frac{t_{k+1}}{w}} f(u)du\right], \quad x \in \mathbb{R} \tag{1}$$

where $f: \mathrm{R} \to \mathrm{R}$ is a locally integrable function, such that the above series converges for each $x \in \mathrm{R}$ and $\{t_k\}_{k\in Z}$ is a sequence of real numbers such that $-\infty < t_k < t_{k+1} < \infty$ for every $k \in \mathrm{Z}$, $\lim_{k\to\pm\infty} t_k = \pm\infty$ and there are two positive constants Δ, δ such that $\delta \leq t_{k+1} - t_k \leq \delta$. Taking $\Delta_k = t_{k+1} - t_k$ for every $k \in Z$, with a suitable kernel function $\chi \in L^1(R) \cap C(R)$.

In order to reconstruct signals or images, one need to use a nonlinear process. So, the theory of nonlinear operators becomes crucial. The nonlinear Kantorovich sampling type series is of the form

$$S_w f(x) = \sum_{k=-\infty}^{k=\infty} \chi\left(wx - t_k\right)\left[\frac{w}{\Delta_k}\int_{\frac{t_k}{w}}^{\frac{t_{k+1}}{w}} f(u)du\right], \quad x \in R \tag{2}$$

where $\chi : \mathrm{R}^2 \to \mathrm{R}$ is a kernel functional satisfying suitable properties.

In Signal Processing, more information is usually known in a neighborhood of a point rather than exactly at that point. As a matter of fact, the so-called jitter error occurs when one cannot match exactly the values $f\left(\frac{t_k}{w}\right)$ and therefore the above average approach reduces jitter errors.

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The theory in the multivariate setting is important from the point of applications, indeed in signal theory and image processing. The family of operators we take into consideration in [5] are of the form

$$\left(S_w^{\chi} f\right)(\underline{x}) = \sum_{\underline{k}\in\mathbb{Z}^n} \chi\left(w\underline{x} - t_{\underline{k}}\right)\left[\frac{w^n}{A_{\underline{k}}} \int_{\mathbb{R}_{\underline{k}}^{w}} f(\underline{u})d\underline{u}\right], \ \underline{x}\in\mathbb{R}^n \tag{3}$$

where $f:\mathbb{R}^n \to \mathbb{R}$ is a locally integrable function such that the above series is convergent for every $\underline{x}\in\mathbb{R}^n$. Here $\chi:\mathbb{R}^n \to \mathbb{R}$ is a kernel function satisfying suitable properties and $t_{\underline{k}} = \left(t_{k_1}, t_{k_2}, \ldots, t_{k_n}\right)$ is a vector, where $\left(t_{k_i}\right)_{k_i\in\mathbb{Z}}$, $i = 1, 2, \ldots, n$ is a sequence of real numbers with some properties. Also, $\mathbb{R}_{\underline{k}}^{w} = \left[\frac{t_{k_1}}{w}, \frac{t_{k_1+1}}{w}\right] \times \left[\frac{t_{k_2}}{w}, \frac{t_{k_2+1}}{w}\right] \times \ldots \times \left[\frac{t_{k_n}}{w}, \frac{t_{k_n+1}}{w}\right]$, $w > 0$ and $A_{\underline{k}} = \Delta_{k_1} \cdot \Delta_{k_2} \cdot \ldots \cdot \Delta_{k_n}$ with $\Delta_{k_i} = t_{k_i+1} - t_{k_i}$, i = 1, 2, ..., n. This linear multivariate setting allowed to work with discontinuous signals, treating in this way a problem not covered by the classical theory, which considers only continuous functions.

In image processing, to have at disposal a theory for nonlinear sampling Kantorovich operators; indeed, in some instances, in order to reconstruct signals or images, one need to use a nonlinear process. The family of nonlinear multivariate sampling Kantorovich operators, here considered, is of the form

$$\left(S'^{\chi}_{w} f\right)(\underline{x}) = \sum_{\underline{k}\in\mathbb{Z}^n} \chi\left(w\underline{x} - t_{\underline{k}}, \frac{w^n}{A_{\underline{k}}} \int_{\mathbb{R}_{\underline{k}}^{w}} f(\underline{u})d\underline{u}\right), \ \underline{x}\in\mathbb{R}^n \tag{4}$$

where $\chi:\mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is a kernel function.

Literature Review

Bardaro[3] in the year 2007, proposed a concept of generalized sampling version of Kantorovich operators as in equation (1) in Orlicz space which has an application in processing of discontinuous signals. It is the first one to be primarily concerned with the Kantorovich version of generalized sampling series. The use of images in medicine, is very important under various aspects, for in- stance, to make diagnosis and to effect surgical operations.

Applications are given to several sampling series with special kernels, especially in the instance of discontinuous signals. Graphical representations for the various examples are included in the same paper. In particular, they study algorithms based on sampling operators of the Kantorovich type to enhance endo-vascular images. This research was developed in collaboration with the doc- tors of the department of vascular surgery of the "Santa Maria della Misercodia" hospital of Perugia.

Vinti and Zampogni[4] in 2009, introduced a nonlinear version of the Kantorovich sampling type series in a nonuniform setting in equation (2). This helped them to reconstruct signals (functions) which are continuous or uniformly continuous. Moreover, they studied the problem of the convergence in the setting of Orlicz spaces, this allowed them to treat signals which are not necessarily continuous.

The theory of nonlinear integral operators has been studied in various papers by Musielk[1] and in extensively developed in the monograph by Bardaro, Musielak and Vinti[2]. One of the main problems to be solved in passing from the linear to the nonlinear setting (when one must deal with a nonlinear integral or discrete operator) is that of introducing a suitable notion of singularity for the family of kernel functions. This notion of singularity should reduce to the classical one for linear operators. A hypothesis reflecting a suitable notion of singularity was first introduced by in modular spaces and then weakened in later. It was successively extended in the setting of compact abelian groups and again further developed.

Another problem which arises in connection with estimates and convergence results for non- linear operators is what kind of assumption one must impose on the kernel function and, in this respect, a kind of Lipschitz condition on the kernel function must be assumed. This last condition is always used in the literature in order to deal with approximation by means of nonlinear integral (or discrete) operators. The assumptions used in order to solve the above problems seem to be the most reasonable ones that reproduce the classical ones in the particular case of linear operators.

As a consequence, the methods and the results in the nonlinear setting are different from those of the linear frame. To give an idea of the difference between the two theories, they observed that the regularity results which hold in the linear case are no longer valid in the nonlinear case. More- over, behind the mathematical significance of the nonlinearity of equation (2), there is a meaning in terms of its applications in Signal

Processing. Indeed, the operators (2) are suitable in order to describe nonlinear transformations generated by signals that, during their filtering process, produce new frequencies. In the paper[4], where operators (2) are introduced for the first time, they first give pointwise and uniform convergence results for the operators $S'_w f$ towards f, as $w \to \infty$. Successively, they studied the convergence of $S'_w f$ to f (as $w \to \infty$) in the setting of Orlicz spaces. Concerning applications, they first show that their theory applies to L^p-spaces, interpolation spaces ($L^\alpha \log^\beta L$-spaces) and exponential spaces.

The setting of L^p-spaces is particularly important from the point of view of the applications in Image Processing, since, as is well known, discontinuities represent jumps of grey-levels which occur in the contours or in the edges of images. As an example, a faithful reconstruction of contours and edges is very important in medical diagnostics.

In 2011[5], Costarelli and Vinti worked on linear version of the Sampling Kantorovich type operator in a multivariate setting as in equation (3) and showed its application to image processing. By means of these operators they were able to reconstruct continuous and uniformly continuous signals/images (functions). The study of modular convergences of these operators in the setting of Orlicz space $L^\phi\left(\mathbb{R}^n\right)$ allowed them to handle the case of not necessarily continuous signals/images. The convergence theorems in $L^p\left(\mathbb{R}^n\right)$−spaces, $L^\alpha \log^\beta L\left(\mathbb{R}^n\right)$-spaces and exponential spaces follow as particular cases.

Multivariate setting is important from the point of applications; indeed, in signal theory, in order to deal with image processing. The paper[5], showed the concrete examples showing how the theory can be applied to image approximation and that approach allowed them to work with discontinuous signals. The importance of this fact, release just in the multivariate setting, where the possibility of approximating images in case of discontinuous functions means to detail the contours of the image itself, since discontinuities represent jumps of grey levels that imply high contrast. Therefore, this theory became very important when especially with image enhancement in case of biomedical images where the shape of the contours can suggest some specific pathology.

In 2013[6], the study on a nonlinear version of the sampling Kantorovich type operators in a multivariate setting was done. By means of the above operators defined as in equation (4), they were able to reconstruct continuous and uniformly continuous signals/images (functions). Moreover, the modular convergence of these operators in the setting of Orlicz spaces allowed to deal the case of not necessarily continuous signals/images. The convergence theorems in $L^p\left(\mathbb{R}^n\right)$-spaces, and exponential spaces follow as particular cases. Graphical representations for the various examples and image processing showed that the problem of reconstructing an image at the discontinuity points means to be able to describe in detail the edge or the contours of the image where jumps of gray levels occur.

It is observed that the order of reduction of error depends on the kernels of operators and the engaged functions. Several examples of kernels are considered in all these papers, typically used in approximation theory for application to image and signal processing.

Conclusion

Medical images in general are not linear. Multivariate sampling Kantorovich operators are suitable for studying not necessarily continuous signals/images as the ones involved in the medical eld. The construction of multivariate kernel affects the order of reduction in error when uniform and modular approximation are considered. Convergent results for such a family of discrete operators translate into reconstruction and even enhancement of a given signal/image increasing the sampling rate.

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